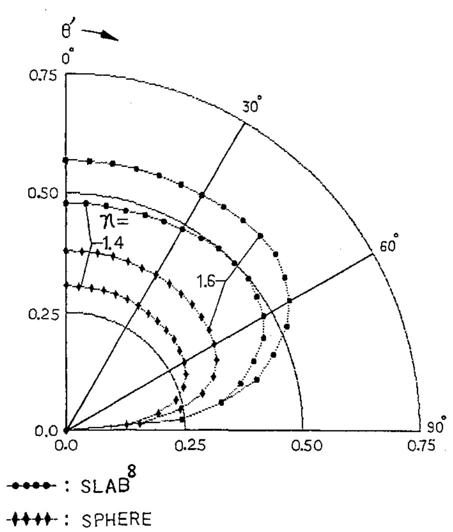

Fig. 3 Comparisons of directional emittance for different geometries at $\omega = 0.95$ (optical thickness = 5.0 for the slab and optical diameter = 5.0 for the sphere).

Figure 2 shows that the refractive index at which the maximum emittance occurs increases with the increase in albedo. This complicated behavior is because of the combined influence of scattering and boundary reflection. This is consistent with the results for planar[8] and cylindrical[5] geometries. Besides, the directional emittance decreases with the increase in the albedo and the optical radius. Such behaviors are also consistent with those for planar and cylindrical geometries.

Since the planar medium is infinite in all directions parallel to the boundaries, the cylindrical medium is infinite only in the axial direction, and the spherical medium is finite in all directions, we can expect that the directional emittance of a sphere is smaller than that of a slab[8] or that of a cylinder[5] with the same physical parameters. The normal emittance of a sphere with $n = 1.6$ is about two-thirds as large as that of a slab with the same refractive index, and the difference between the normal emittance of a slab and that of a sphere increases with the decrease in refractive index, as shown in Fig. 3. This is because the influence of geometry decreases with the increase in surface reflectivity.

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Variable Specific Heat and Thermal Relaxation Parameter in Hyperbolic Heat Conduction

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Introduction

THE conduction of heat in solids is usually treated mathematically as a diffusion process in which the effect of a thermal disturbance is transmitted throughout the solid with an infinite velocity. However, in some situations, primarily those involving extremely short times or temperatures near absolute zero, the mode of heat conduction is not diffusive (parabolic) but propagative (hyperbolic).[1-3] In the present paper, both the specific heat and the thermal relaxation parameter of the medium are studied parametrically. The effect of varying the specific heat is investigated by assuming a linear relationship with temperature. In addition to studying the effects of a linear specific heat, the thermal relaxation parameter is modeled as a function of time. The relaxation parameter determines how dominant the hyperbolic nature of the heat conduction is. If the relaxation parameter is zero, the hyperbolic heat conduction (HHC) equation reduces to the parabolic equation, and the energy transport is diffusive. As the relaxation parameter increases, the hyperbolic term becomes more important, and the thermal propagation speed decreases. MacCormack's predictor-corrector scheme was used to solve the HHC problems, and an implicit Crank-Nicolson scheme was used to solve the parabolic problems.

Formulation

The medium was taken to be a slab of dimensionless thickness $\eta = 1$. The density and thermal conductivity were assumed to be constant, and the specific heat was assumed to be a function of the local temperature, and the thermal relaxation parameter was assumed to be a function of time. The HHC equation results from the use of the non-Fourier heat flux equation given in dimensionless form as

$$\tau \frac{\partial Q}{\partial \xi} + 2Q + K \frac{\partial \theta}{\partial \eta} = 0 \tag{1}$$

where Q is the dimensionless conduction heat flux, θ is the dimensionless temperature, τ is the dimensionless thermal relaxation parameter, and K is the dimensionless thermal conductivity. In addition, the dimensionless distance and time are given, respectively, by η and ξ. Clearly, when the relaxation parameter τ is equal to 0, the non-Fourier heat flux equation reduces to the classical Fourier heat flux equation. Although the heat flux equations are different for the hyperbolic and parabolic formulations, the energy equation remains un-

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changed for both models and is given in dimensionless form as

$$\frac{\partial \theta}{\partial \xi} + \frac{1}{RC}\frac{\partial Q}{\partial \eta} = 0 \tag{2a}$$

where R is the dimensionless density and C is the dimensionless constant pressure specific heat. In the hyperbolic case, the enthalpy formulation of the energy equation is used due to the inclusion of variable specific heat, and the energy equation thus takes the form

$$\frac{\partial H}{\partial \xi} + \frac{\partial Q}{\partial \eta} = 0 \tag{2b}$$

where H is the dimensionless enthalpy and is defined as

$$H = \int_0^\theta R(\theta')\, C(\theta')\, \mathrm{d}\theta' \tag{2c}$$

For the diffusion, or parabolic problem, the heat conduction equation obtained from the heat flux and energy equations is given by

$$\frac{\partial \theta}{\partial \xi} + \frac{1}{2RC}\frac{\partial}{\partial \eta}\left[K\frac{\partial \theta}{\partial \eta}\right] \tag{3}$$

MacCormack's predictor-corrector scheme, having previously been validated for HHC, was chosen for the present hyperbolic analysis.[3] The Crank-Nicolson method[4] was used to integrate the parabolic heat conduction equation.

Results and Discussion

In the parametric study which follows, hyperbolic and parabolic solutions to the one-dimensional heat conduction equations were determined for a finite slab with a dimensionless thickness $\eta = 1$, a heat flux $Q = 1$ applied at the left surface, and an adiabatic boundary condition at the right surface. The specific heat was taken to be a linear function of the local temperature, and the thermal conductivity and density were assumed constant, thus $K = 1$ and $R = 1$. In addition to studying the specific heat, the thermal relaxation parameter was also studied and was modeled as an exponential function of time. In all of the hyperbolic numerical calculations, oscillations appeared due to a dispersive effect of the odd derivative error terms in the numerical solution. These oscillations could be removed by numerical smoothing but at the cost of a reduction in the accuracy of the wave front definition.

Specific Heat

The specific heat was taken in dimensionless form to be $C = (1 + \beta_c \theta)$. The values of the coefficient multiplying the temperature (β_c) were taken to be ± 0.2. The results for $\beta_c = 0$ lie between the $\beta_c = \pm 0.2$ curves, and were thus not shown on the figure in order not to crowd the figure. For the results presented here, the thermal relaxation parameter τ was set to unity. Figure 1 shows the temperature vs position at $\xi = 1$ for $\beta_c = \pm 0.2$ for both the hyperbolic and parabolic models. In addition to the expected effect on the temperatures, the velocity of the hyperbolic wave was also affected by a temperature dependent specific heat. As the specific heat increased with temperature, the velocity of the thermal front decreased and vice versa. The wave front for $\beta_c = -0.2$ has reflected from the back surface, and for $\beta_c = 0.2$, the thermal front has not reached the back surface. This inverse relationship between the specific heat and the wave speed can be confirmed by examining the energy equation, where the coefficient in front of the spatial derivative represents the wave speed in the hyperbolic equation. The term multiplying the spatial derivative in Eq. (2a) is $(1/RC)$. Thus the inverse relationship between specific heat and wave speed is both predicted by the energy equation and observed in the numerical solution.

Thermal Relaxation Parameter

The thermal relaxation parameter τ is related to the short time required for the steady-state heat flux to be reached when a temperature gradient is suddenly introduced in a medium. As τ approaches zero, the thermal propagation speed approaches infinity, and the parabolic heat conduction equation is obtained. In Fig. 2, the internal temperatures are shown at dimensionless time $\xi = 1$ for values of $\tau = 0$, 0.5, 1, and 10. The temperatures in the slab are quite large behind the thermal front for $\tau = 10$, but the thermal front is moving at a much slower velocity than for $\tau = 1$. The solution for $\tau = 10$ has large oscillations at the wave front which do not appear in the $\tau = 1$ solution due to numerical restrictions on the Courant number. The thermal front with $\tau = 0.5$ has reflected from the back surface at time $\xi = 1$ and is propagating in the negative direction. Though a small thermal front is observed in the $\tau = 0.5$ solution, the temperature distribution is beginning to resemble the parabolic solution. An energy balance on the slab for different values of τ demonstrates that the energy stored in the medium is the same regardless of the value of τ. Thus, for large values of τ, large temperatures are observed over a small region (as a result of a slow thermal front velocity), and as τ approaches zero, front velocities approaching infinity and temperatures asymptotically approaching the parabolic solution are ob-

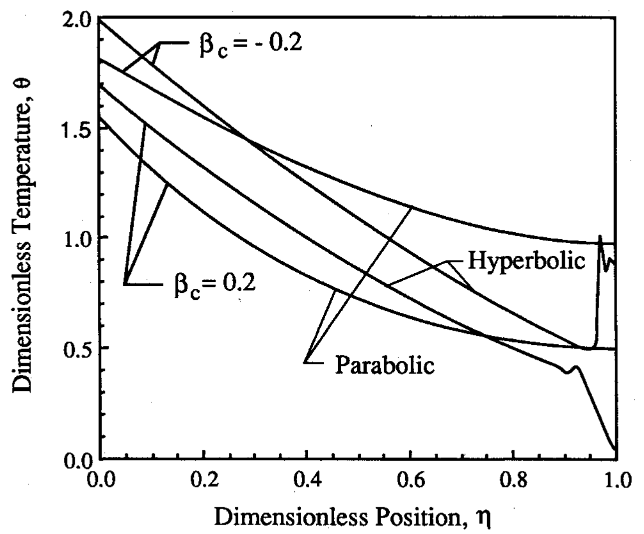

Fig. 1 Effect of variable specific heat on internal temperature at $\xi = 1$ with $\tau = 1$.

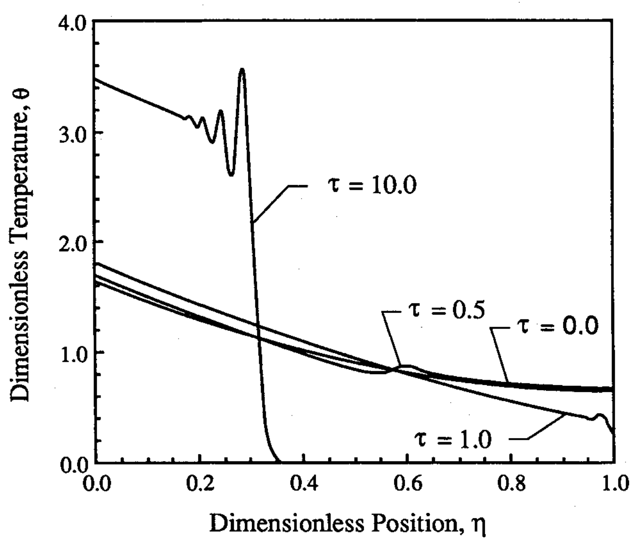

Fig. 2 Effect of relaxation parameters on internal temperature at $\xi = 1$.

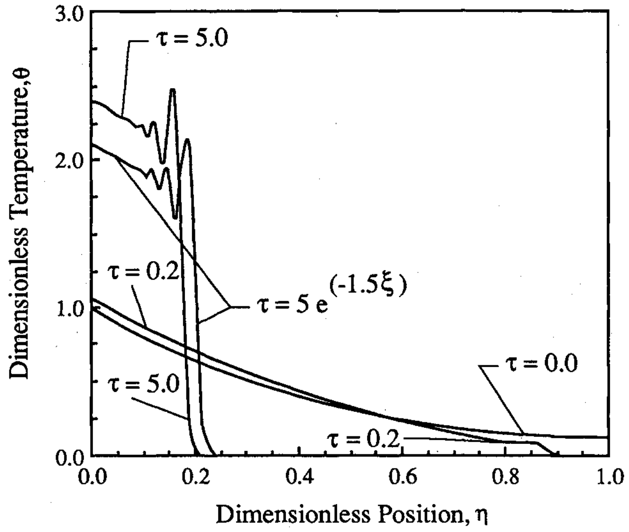

Fig. 3 **Effect of variable relaxation parameter on internal temperature at $\xi = 0.4$.**

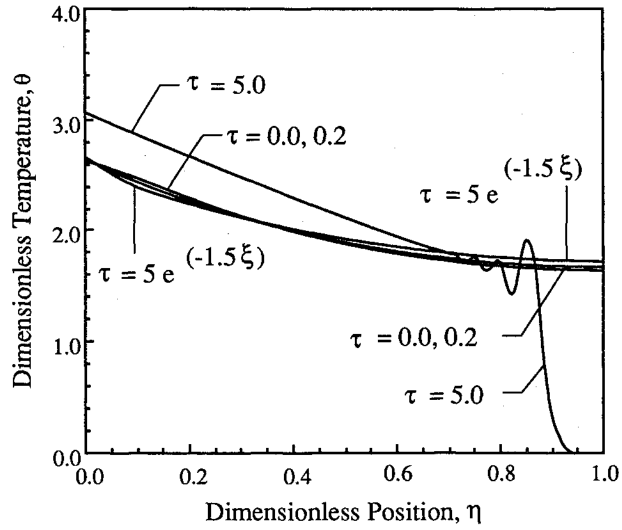

Fig. 4 **Effect of variable relaxation parameter on internal temperature at $\xi = 2$.**

served. The steady-state solutions for both the hyperbolic and parabolic cases are the same as is obvious from the fact that the only difference in the governing equations is a transient term. Thus, the hyperbolic solution tends toward the parabolic solution as time increases. As can be seen in Fig. 2, as τ decreases, this convergence between the two solutions occurs more rapidly.

In the above solution, the relaxation parameter was treated as a constant. However, it may sometimes be necessary to consider a situation which is strongly hyperbolic near time zero (i.e. unusually large temperatures obtained extremely rapidly near the surface), but the energy quickly diffuses into the medium as would be expected in the parabolic model. In this case, it can be postulated that a decreasing thermal relaxation parameter may provide a mechanism to avoid what appears to be nonphysical internal temperatures, and at the same time, allows a finite thermal propagation speed and large surface temperatures. Though the thermal relaxation parameter is most likely a function of temperature and/or heat flux, a time-dependent relaxation parameter is able to model the decreasing effect needed to speed convergence to the parabolic solution. The effect of taking this approach is shown in Figs. 3 and 4. Figure 3 shows the temperatures in the medium at an early time of $\xi = 0.4$ for four different values of τ, $\tau = 0$, 0.2, 5, and $5e^{-1.5\xi}$. It can be seen in the figure that the curves for $\tau = 5$ and $5e^{-1.5\xi}$ resemble each other both in magnitude and shape. This is due to the fact that at small times, the exponential term has little effect on τ. The curves for $\tau = 0$ and 0.2 also resemble each other since the value of $\tau = 0.2$ results in a relatively large front velocity; although a slight, but noticeable, wave front still remains for the $\tau = 0.2$ curve. Figure 4 shows the temperatures for the same cases at a later time of $\xi = 2$. At this time, the curve for $\tau = 5$ still retains its hyperbolic nature with a slowly moving thermal front. As expected, there is still little difference between the $\tau = 0$ and 0.2 solutions. However, the temperature distribution for $\tau = 5e^{-1.5\xi}$ is now similar to those with $\tau = 0$ and 0.2, due to the exponentially decreasing relaxation parameter. Thus, it can be seen that a time-dependent relaxation parameter is able to rapidly "bridge the gap" between a strongly hyperbolic solution at early times and a parabolic solution at later times.

As was discussed earlier when analyzing the thermal properties, the effects observed numerically can be readily verified by examination of the non-Fourier heat flux equation. The coefficient term multiplying the spatial derivative in Eq. (1) is K/τ, which implies an inverse relationship between τ and the thermal propagation speed.

Though the Crank-Nicolson method was used to solve the parabolic equation, the solution could also be obtained by setting the relaxation parameter to be quite small and solving the hyperbolic equations. Numerical stability of MacCormack's method, however, will dictate how much τ can be reduced. Setting the relaxation parameter to zero eliminates the time-dependent flux equation from MacCormack's scheme and implies a different numerical technique to be used due to the different nature of the equations involved. In addition, the use of two different numerical schemes to arrive at similar solutions ($\tau = 0$ and $\tau << 1$) is an excellent check on the numerical schemes.

Concluding Remarks

Numerical solutions were obtained for a finite slab with an applied surface heat flux at the left boundary and an adiabatic right boundary using both the hyperbolic and parabolic heat conduction equations. The effects on the solutions of varying both the specific heat and thermal relaxation parameter were observed. In the hyperbolic solutions, as the specific heat and thermal relaxation parameter were observed. In the hyperbolic solutions, as the specific heat decreased with temperature, both the temperatures within the medium and the thermal front velocity increased. The inverse was found to be the case if specific heat increased with temperature. The value taken for the thermal relaxation parameter was found to determine the "hyperbolicity" of the heat conduction model. For a small relaxation parameter, the thermal wave velocity approached infinity, and the hyperbolic solution resembled the parabolic solution. For large values, the temperatures in the medium behind the thermal front were large compared with the parabolic solution, and the thermal front propagated quite slowly. The use of a time-dependent relaxation parameter produced solutions where the thermal energy propagated as a high-temperature wave initially but approached a diffusion process more rapidly than was possible with a large constant relaxation parameter. Thus, a time decaying thermal relaxation parameter is able to accommodate the physical aspects of both the hyperbolic and parabolic models, i.e., finite propagation speeds and high surface temperatures but yet enough diffusion to eliminate excessively high internal temperatures that may result from summation or reflection of thermal waves. Since for short time applications, some form of the hyperbolic solution is expected to give the most accurate solution, the use of a time-dependent relaxation parameter may assist in the development of such a model.

Acknowledgment

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